Math 43 Midterm 2 Review

[1] Eliminate the parameter to find rectangular equations corresponding to the following parametric equations. For [a][d][e], write y as a function of x.

[a]
$$x = \frac{t}{1-t}$$

 $y = \frac{t-1}{1+t}$
[b] $x = 3+5\tan t$
[c] $x = 8+6\cos t$
[d] $x = 5\ln 4t$
 $y = 7-\sin t$
[d] $y = 2t^3$

[e]
$$x = e^{3t}$$
 [f] $x = \cos 2t$
 $y = e^{-t}$ $y = 2\cos t$

- [2] AJ is standing 24 feet from BJ, who is 5 feet tall. AJ throws a football at 30 feet per second in BJ's direction, at an angle of 60° with the horizontal, from an initial height of 6 feet.
 - [a] Write parametric equations for the position of the football.
 - [b] Does the football hit BJ, go over BJ's head, or hit the ground before reaching BJ?
- [3] Find parametric equations for the following curves using templates from your lecture notes, textbook and exercises.
 - [a] the line through (-3, -6) and (7, -2)
 - [b] the circle with (-3, -6) and (7, -2) as endpoints of a diameter
 - [c] the portion of the graph of $y = 2x^4 3x^3 + 1$ from (-1, 6) to (2, 9)

[4] Find the value of
$$\sum_{n=3}^{8} (-1)^n n(n-4)$$
.

[5] Write the repeating decimal $0.4\overline{72}$ as a simplified fraction. <u>NOTE: Only the 72 is repeated.</u>

[6] Calculate
$$\begin{pmatrix} 200\\ 4 \end{pmatrix}$$
.

- [7] Use sigma notation to write the series $\frac{1}{7 \cdot 3} + \frac{1}{4 \cdot 6} + \frac{1}{1 \cdot 12} \frac{1}{2 \cdot 24} \dots \frac{1}{17 \cdot 768}$.
- [8] Find the coefficient of x^{34} in the expansion of $(2x^5 3x^2)^{11}$.
- [9] Find the value of $\sum_{n=3}^{\infty} 4(0.97)^{2n-1}$. <u>HINT: Write out the first few terms first.</u>
- [10] Find the first 5 terms of the sequence defined recursively by $a_n = 2a_{n-1} 3$, $a_1 = 4$. Is the sequence arithmetic, geometric or neither ? Explain how you arrived at your conclusion.
- [11] Use Pascal's triangle and the Binomial Theorem to expand and simplify

[a]
$$(3x-2y)^6$$
 [b] $(\sqrt{x}-\frac{2}{x})^4$

[12] EJ bought a new car in 1998. The registration fee was \$800 that year. Each year, the registration fee decreased by 10%. The car was eventually sold for scrap in the year when its registration fees were \$3.34. What year was EJ's car sold for scrap?

- CJ and DJ both just graduated from college and started new jobs. Neither could afford the market rate for apartment rentals, so they [13] worked out deals with their landlords. CJ agreed to pay \$400 rent the first month, and each month after, \$7 more rent than the previous month. DJ agreed to pay \$380 rent the first month, and each month after, 2% more rent than the previous month. After 2 years, who will have paid more rent altogether, and by how much ?
- Prove by mathematical induction: [14]

[a]
$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$
 [b] $\sum_{i=0}^n (2i+1)3^{i-1} = \frac{1+n3^{n+1}}{3}$ for all integers $n \ge 1$ for all integers $n \ge 0$

[c]
$$a + ar + ar^{2} + \dots + ar^{n} = \frac{a(r^{n+1} - 1)}{r - 1}$$
 [d] $\sum_{i=1}^{n} \frac{3}{(i+3)(i+2)} = \frac{n}{n+3}$

for all integers $n \ge 1$

$$\sum_{i=1}^{n} \frac{3}{(i+3)(i+2)} = \frac{n}{n+1}$$

for all integers $n \ge 1$

- Find the sum of the series $-73 66 59 52 \dots + 529$. [15]
- Without graphing (or using your calculator), describe the difference between the curves with parametric equations [16] $x = 1 - t^4$ $x = 1 - e^{t}$ $x = 1 - \ln t$ $x = 1 - \sin t$ $y = t^4$, $y = e^t$, $y = \ln t$, and $y = \sin t$

Answers

[1]	[a]	$y = \frac{-1}{2x+1}$	[b]	$\frac{(y-4)^2}{4}$	$\frac{(x-3)^2}{25} = 1$	[c]	$\frac{(x-8)^2}{36} + (y-7)^2 = 1$
	[d]	$y = \frac{1}{32}e^{\frac{3x}{5}}$	[e]	$y = x^{-\frac{1}{3}}$		[f]	$x = \frac{y^2}{2} - 1$
[2]	[a]	$x = 15t$ $y = 6 + 15\sqrt{3} t - 16t^2$	[b]	over BJ's he	ad		
[3]	[a]	x = -3 + 10t $y = -6 + 4t$	[b]	$x = 2 + \sqrt{2}$ $y = -4 + \sqrt{2}$	$\frac{19}{29}\cos t$ $\sqrt{29}\sin t$	[c]	x = t $y = 2t^4 - 3t^3 + 1$ $t \in [-1, 2]$
[4]	21						
[5]	$\frac{26}{55}$						
[6]	64,684,950						
[7]	$\sum_{n=1}^{9} \frac{1}{3(10-3n)(2)^{n-1}}$						
[8]	-11,547,360						
[9]	58.1207						
[10]	4, 5, 7, 11, 19 neither arithmetic nor geometric						
[11]	[a] $729x^6 - 2916x^5y + 4860x^4y^2 - 4320x^3y^3 + 2160x^2y^4 - 576xy^5 + 64y^6$						
	[b] $x^2 - 8x^{\frac{1}{2}} + 24x^{-1} - 32x^{-\frac{5}{2}} + 16x^{-4}$						
[12]	2050						
[13]	DJ will have paid \$28.31 more rent						
[14]	PROOFS INCLUDED IN UPCOMING SOLUTIONS HANDOUT						
[15]	19,836						

[16] ANSWERS & EXPLANATIONS INCLUDED IN UPCOMING SOLUTIONS HANDOUT